

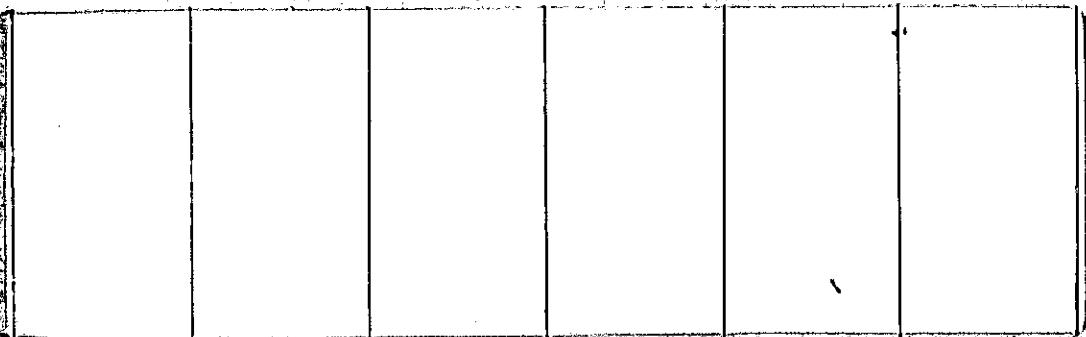
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TURBULENT BOUNDARY LAYER SEPARATION ON  
MULTI-ELEMENT INFINITE SWEEP WINGS (Flow  
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FLOW RESEARCH NOTE NO. 70

THE SIMULATION OF TURBULENT BOUNDARY LAYER SEPARATION ON  
MULTI-ELEMENT INFINITE SWEPT WINGS\*

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MARCH, 1975

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THE SIMULATION OF TURBULENT BOUNDARY LAYER SEPARATION ON  
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F. A. Dvorak and E. W. Geller

INTRODUCTION

A computer program (acronym VIP) which determines the viscosity-dependent potential flow and boundary layer developments over two-dimensional and infinite swept multi-element wings is currently in use at the Large Scale Aerodynamics Branch of NASA Ames Research Center (ref. 1). The analyses used in the program are presently valid for fully-attached flow, although small amounts of turbulent boundary layer separation can be accommodated. In many high lift applications, the amount of turbulent separation is not small, and it has a considerable effect on the potential flow. Therefore, some improved method of simulating the effect of separation is desirable, and developing this method was the goal of the work reported here.

Two types of turbulent boundary layer separation are usually present in high lift applications. The first is that associated with trailing edge stall. In this case separation first occurs at the trailing edge of an element and progresses forward as the angle-of-attack increases, producing a separation bubble which extends downstream of the trailing edge. The second type of separation is that which often occurs in the cove region between the main wing element and the flap. This type of separation can effect the theoretical predictions, even at low angles of attack. The separation bubble in this case is generally closed.

Theoretical modeling of separation has been attempted by several authors, and, at least in two-dimensional flow, separated flows have been modeled with some degree of success (refs. 2 and 3). A simple yet effective method has been demonstrated in three dimensions (ref. 4). All of these methods use criteria developed from boundary layer theory for prediction of the separation point. Representation of the separation region in the potential flow by each method differs basically only in detail. Each of these methods uses fluid

outflow from the surface in the separated region to give a displacement effect similar to the real wake. None of the methods calculate the pressure in the separation region directly, but rely on some criterion to determine the separated flow pressure level. However, it must be said that, in general, the methods predict the upstream pressure distributions in a satisfactory manner once a suitable outflow distribution has been found.

#### THE MATHEMATICAL MODEL

The flow external to the boundary layer and the separated wake is irrotational (i.e. vanishing curl) and for low Mach numbers, nearly solenoidal (i.e. vanishing divergence), and hence, it is treated as a potential flow. Replacing the boundary layer and the separated wake by an extension of this potential flow (i.e. an analytic continuation) into the body surface gives a flow which has a nonzero normal component at the surface; that is, it has outflow at the surface. Thus, the boundary layer and the separated wake in the real case are replaced by fluid originating from the body surface as shown in fig. 1. It is this potential flow solution that is sought, and the problem is well-posed mathematically if the normal component of velocity is prescribed at the body surface and if the uniform stream at infinity is prescribed. Rules governing the outflow distribution which accounts for boundary layer growth upstream of separation are well established and form the basis of the computer program, VIP. Obtaining rules for the outflow distribution that displaces the potential flow in the same way as the separated wake was the main consideration of the work reported here. For the following, outflow is defined to be the normal component of velocity at the body surface.

#### TRAILING EDGE SEPARATION

##### Approximating the Outflow Distribution

As indicated above, an outflow distribution for the potential flow model does exist such that the potential flow part of the real flow is duplicated. Rather than find the exact outflow distribution in the separated region, we seek the member of the two-parameter family shown in fig. 2 that gives the best approximation. The outflow increases linearly with the distance downstream from the separation point up to a certain place (called the knee point) after which it

is constant. The two parameters prescribing a member of the family are  $\Delta q_{\max}$ , the maximum increase in outflow from the value at separation, and  $r$ , the ratio of the distance to the knee point and the distance to the trailing edge, both measured from the separation point along the airfoil surface. The parameter  $\Delta q_{\max}$  fixes the magnitude, and the parameter  $r$  determines the shape of the distribution.

The acceptability of this approximation is demonstrated in fig. 3. The figure gives the results of a study for which  $r$  and  $\Delta q_{\max}$  were varied to obtain a potential flow solution giving the best agreement with experimental data (taken from ref. 5) for a NACA 2412 airfoil at a  $19^{\circ}$  angle-of-attack. Boundary layer growth upstream of separation was neglected for this study.

Before the study was made, it was anticipated that the required distribution could not be approximated well enough by the crude representation depicted in fig. 3, and that perhaps a different shape would be required, especially for the initial ramp. The precise agreement with experiment was welcome.

#### The Governing Algebraic Equations for the Potential Flow Problem

The potential flow is constructed as the sum of the free-stream and the velocity fields associated with a vorticity and a source sheet on the surface of the airfoil. The equations for the solution of the potential flow problem were developed in reference 1 for the case of no separation. When modified for separation modeling according to the preceding ideas, the equations expressing the surface boundary conditions become

$$\sum_{j=1}^N a_{ij} \gamma_j + \left( \sum_{j=1}^N b_{ij} f_j \right) \Delta q_{\max} = -\sin(\alpha - \delta_1) - \sum_{j=1}^N b_{ij} q_j, \quad i = 1, N-1 \quad (1)$$

where   
 $a_{ij}$  = vortex influence coefficients  
 $b_{ij}$  = source influence coefficients  
 $\gamma_j$  = unknown vortex strengths  
 $q_j$  = source strength associated with boundary layer growth\*  
 $f_j$  = source weighting function  $f$  (see fig. 2)\*\*

---

\* In the separated region this is taken to be  $q_{sep}$ , the value at the separation point, and the total source strength is  $q_{sep} + f_j \Delta q_{\max}$  (see fig. 2).

\*\*  $f_j$  is defined to be zero for points outside the region of separation

- $\Delta q_{\max}$  = unknown scaling factor for the source increment associated with separation (see fig. 2)  
 $\alpha$  = angle of attack  
 $\delta_i$  = surface inclination.

The Kutta condition equations are also modified from those occurring in ref. 1. They become

$$\gamma_u \sin \theta - \Delta q_{\max} \cos \theta = q_l + q_u \cos \theta \quad (2a)$$

$$\gamma_l \sin \theta + \Delta q_{\max} \cos \theta = -(q_u + q_l \cos \theta) \quad (2b)$$

where the subscripts  $u$  and  $l$  imply upper and lower surface trailing edge evaluation respectively, and  $\theta$  is the trailing edge angle. Note that  $q_u$  is taken to be the value of the source strength due to boundary layer growth at the separation point (consistent with the definition of  $q_j$  in the separated region).

In both equations (1) and (2) the contribution of the separation model ( $\Delta q_{\max}$ ) to the source distribution appears distinctly (on the left-hand side) from the contribution of the boundary layer growth ( $q_j$ ,  $q_l$ , and  $q_u$ ). This separation occurs when terms with unknowns are placed on the left, and the remaining on the right. In equation (1), for example, the total contribution from the surface source is

$$\sum_{j=1}^N b_{ij} (q_j + f_j \Delta q_{\max}) = \left( \sum_{j=1}^N b_{ij} f_j \right) \Delta q_{\max} + \sum_{j=1}^N b_{ij} q_j. \quad (3)$$

The first term on the right-hand side has the unknown  $\Delta q_{\max}$ , whereas the second term has no unknowns.

#### The Empirical Determination of the Parameter r

Previously, the existence of an outflow distribution which will produce the same pressure distribution upstream of separation as obtained by the separation wake in the real case for a NACA 2412 airfoil was shown. We wish to predict that outflow distribution. In terms of the two-parameter family, which is assumed to be adequate in all cases, we need to predict the values of  $r$  and  $\Delta q_{\max}$ . The method described in the following section allows the prediction of  $\Delta q_{\max}$ , given the value for  $r$ . The value for  $r$  must be supplied from empirical evidence. For the present time, we choose  $r = .115$  on the

basis of calculations for the NACA 2412 airfoil discussed in the section,  
Comparison with Experiment.

#### Solution of the Potential Flow Problem

When the parameter  $r$  is prescribed, the terms  $f_j$  appearing in equation (1) are known, and equations (1) and (2) are  $N + 1$  in number and are linear in the  $N + 1$  unknowns  $\gamma_j$  and  $\Delta q_{\max}$ . Note that  $\gamma_u$  and  $\gamma_l$  are not additional unknowns but are included in the set  $\gamma_i$ . Standard techniques can be applied to solve this square set of equations.

Once the source and vortex strengths are determined, the surface pressures about the configuration can be calculated. The pressure in the separated flow region is determined from the assumption that the pressure everywhere in the separation zone is constant and equal to the pressure at the point of separation.

#### Comparison with Experiment

Application of the above technique to the NACA 2412 airfoil at  $\alpha = 19^\circ$  and with separation prescribed to be at 18% chord was made with various values of  $r$  prescribed. The results, given in fig. 4, show excellent agreement with experiment for  $r = .115$ . From this evidence  $r$  has been set at this value. Thus,  $r$  has been empirically determined from the experimental data on the NACA 2412 airfoil.

When we applied this method to an NACA 23012 airfoil using the same value for  $r$  that was found to give such excellent results for the NACA 2412 airfoil, the answers were unrealistic. A possible explanation of this result is given in the section, Considerations about the Governing Algebraic Equations.

### COVE SEPARATION

#### Approximating the Outflow Distribution

For the cove separation problem, an outflow distribution similar to the one first used by Jacob (ref. 2) was used and is shown in fig. 5. The parameters  $a$  and  $\Delta q_{\max}$  were chosen to be .25 and .2 respectively.

#### The Governing Algebraic Equations

The modification of the equations in reference (1) for this case give

$$\sum_{j=1}^N a_{ij} \gamma_j = -\sin(\alpha - \delta_j) - \sum_{j=1}^N b_{ij} (q_j + \Delta q_j) \quad i = 1, N - 1 \quad (4)$$

and

$$\gamma_u \sin \theta = q_\ell + q_u \cos \theta \quad (5a)$$

$$\gamma_\ell \sin \theta = -(q_u + q_\ell \cos \theta) \quad (5b)$$

where the definitions given for equations (1) and (2) apply, and also

$\Delta q_j$  = the source strength increment associated with separation.

As before, all terms involving unknowns have been placed on the left-hand side. For this separation modeling case, there is no unknown associated with the separation-derived source strength since its distribution has been completely specified.

#### Solution of the Potential Flow Problem

Equations (4) and (5) are not square. There is one more equation than number of unknowns. To complete the system of equations, an additional unknown is provided by adding the influence of a constant strength source distribution just inside the airfoil surface. (A complete discussion of this technique is found in ref. 1, pp. 16 and 17). The addition of the unknown strength source to the N-1 unknown vortex strengths gives a well-conditioned set of equations. As with the trailing edge separation model, the calculated surface pressures in the separation region are meaningless and are replaced by the surface pressure at the calculated separation point.

#### Comparison with Experiment

The potential flow solution just described can be matched with the boundary layer solution by iteration, and the lift and drag can be calculated as described in the section, Viscous/Potential Flow Interaction. This matching procedure and drag calculation is accomplished by an extended version of computer program VIP. Results for application to the RAE 2815 single-slotted flap configuration (ref. 6) are shown in figure 6.\* Particularly at low angles-of-attack, separation in the cove results in experimental lift coefficients lower than expected for fully-attached flow. When the effect of the separation model is included in the calculation method, the predicted lift is in good agreement with experiment.

\* Subsequent to completion of the work on this contract, a small error was discovered in the computer program. As a result, the predicted lift and drag levels are slightly higher than those shown in figure 6. This does not detract, however, from the improvement shown by the inclusion of a separation model.

PROGRAM VIP - THE EXTENDED SEPARATION VERSION

Turbulent Boundary Layer Separation

Many procedures have been developed for the prediction of turbulent boundary layer separation in two-dimensional flow. The most satisfactory of the methods appears to be the approach where separation is predicted when the velocity gradient  $du/dy = 0$ . In the case of integral methods, this translates to the point of vanishing skin friction. Both turbulent boundary layer methods used in program VIP predict separation in a satisfactory manner.

Viscous/Potential Flow Matching Procedure

Once the separation point has been determined by a boundary layer calculation, then a pressure distribution upstream of separation can be calculated by use of the potential flow model described in the preceding. Unfortunately, this pressure distribution is needed as input to the boundary layer program to calculate the separation point. To match the inputs and outputs of these two calculations, an iterative procedure like that presently used in VIP for the boundary layer growth problem is required. The extended version of VIP incorporates this iteration as shown in the flow chart in fig. 7.

Drag Calculation

In VIP the total drag of a configuration is determined by adding the profile drag of the attached boundary layer to the base pressure drag, i.e.,  $C_{D_t} = C_{D_f} + C_{D_p} - C_{P_b}$ , where  $C_{P_b}$  is the pressure coefficient in the separated region. Figure 8 shows a comparison between experiment and theory, where the calculations have been made using the viscous/potential flow interaction analysis method of ref. 1. The results are for the sixth iteration between potential flow, boundary layer and separation model calculations.

#### IMPROVEMENTS TO BE MADE

The extension of VIP that has been described in the preceding should be applied to other airfoils for which experimental data is available to determine whether or not the empiricism introduced using the NACA 2414 data is adequate for airfoils in general. Preliminary results from a NACA 23012 airfoil are unrealistic and suggest that the program is not yet valid. A probable reason for this and two approaches for future work are suggested below.

##### Considerations about the Governing Algebraic Equations

Fundamental arguments indicate that although the set of equations (1) and (2) are square and have been shown to have a solution (by demonstration), the potential flow problem being solved by the set is not well posed. The thrust of the argument is that the Kutta conditions given by equation (2), when combined with the integral equation for which equation (1) is an approximation, provide a well-posed problem if  $\Delta q_{\max}$  is specified. Thus one would expect to be able to specify any value for  $\Delta q_{\max}$  and find a set  $\gamma_i$  which satisfies equations (1) and (2). This capability requires that the equations be linearly dependent since the number of unknowns is less than the number of equations. The equations are not strictly dependent, however, since equation (1) is a numerical approximation to the governing integral equation. Thus, with  $\Delta q_{\max}$  considered to be an unknown, it is possible from a computational stand-point to converge to a solution within a given tolerance.

Why then did the answer happen to be in such good agreement with the experiment for the NACA 2412 as shown in fig. 3 if the solution was the result of a mathematical quirk? The answer is found in the chronology with which the work was developed. The study made to choose the family member of fig. 2 which best approximated experimental data, as shown in fig. 3, was effected by solving equations (1) and (2) with various values for  $r$  as shown in fig. 4. Thus, a good solution of equations (1) and (2) was forced by the choice of  $r$ .

This approach appears legitimate since the purpose was to introduce empiricism to choose  $r$ . A fallacy is indicated, however, by another study which has shown that there are other combinations of  $r$  and  $\Delta q_{\max}$  which fit the experimental data upstream of separation nearly as well as those for fig. 3. This fact is shown in fig. 9 where the results for  $r = .25$  and  $\Delta q_{\max} = .50$  are compared with fig. 3. This second study was accomplished by the use of another

potential flow program, for which any outflow velocity distribution can be specified as input. With this program,  $r$  was fixed at .25, and  $\Delta q_{\max}$  was varied to obtain the best fit with the experiment. As a check, this same program was also used with  $r$  and  $q$  having the values for fig. 3. The results were identical with fig. 3.

The situation appears to be one where the choice of  $r$  is not too critical, and that, for a wide range of values, it is possible to find a value for  $\Delta q_{\max}$  which will give good results. Suppose then that a value of .25 is used for  $r$  in solving equations (1) and (2). The result is shown in fig. 10. The best choice for  $\Delta q_{\max}$ , also shown on the figure, was not obtained in the solution.

It is now apparent that the original choice for  $r$  (namely  $r = .115$ ) was the one for which the computer solution of a nearly dependent set of equations gave the best answer. The choice was not dictated by the physics of separation via empirical matching to an experimental data case (as was originally intended).

#### Future Efforts on the Potential Flow Problem

Two avenues for future work to eliminate the dilemma outlined in the last section are suggested. One approach is to add more empiricism by controlling  $r$  and  $\Delta q_{\max}$  according to rules developed from calculations on all airfoils for which good experimental data can be found. For this approach neither parameter would be obtained as part of the solution to the potential flow problem.

The other approach is to enforce another condition on the potential flow problem based upon the physics of separated flows. An example is the work of Jacob (ref. 2), in which he utilized the existence of nearly constant pressure in the separated wake region. It would be desirable to use some method that could be easily extended to the three-dimensional case (Jacob's work cannot be directly extended since he used the stream function). An outline of such a method follows.

Apparently, a value of  $r$  can be chosen between .1 and .25. The later value is used by Jacob (ref. 2). To make the mathematical problem well posed, we should enforce an additional condition on the flow when the parameter  $\Delta q_{\max}$  is not specified but treated as an unknown. Requiring the pressure to be the same at two points on an estimated position for the separation streamline is suggested. This requirement can be expressed as a linear equation in the unknowns if the direction of the velocity vector is estimated. An iterative procedure to find the separation streamline position may be desirable.

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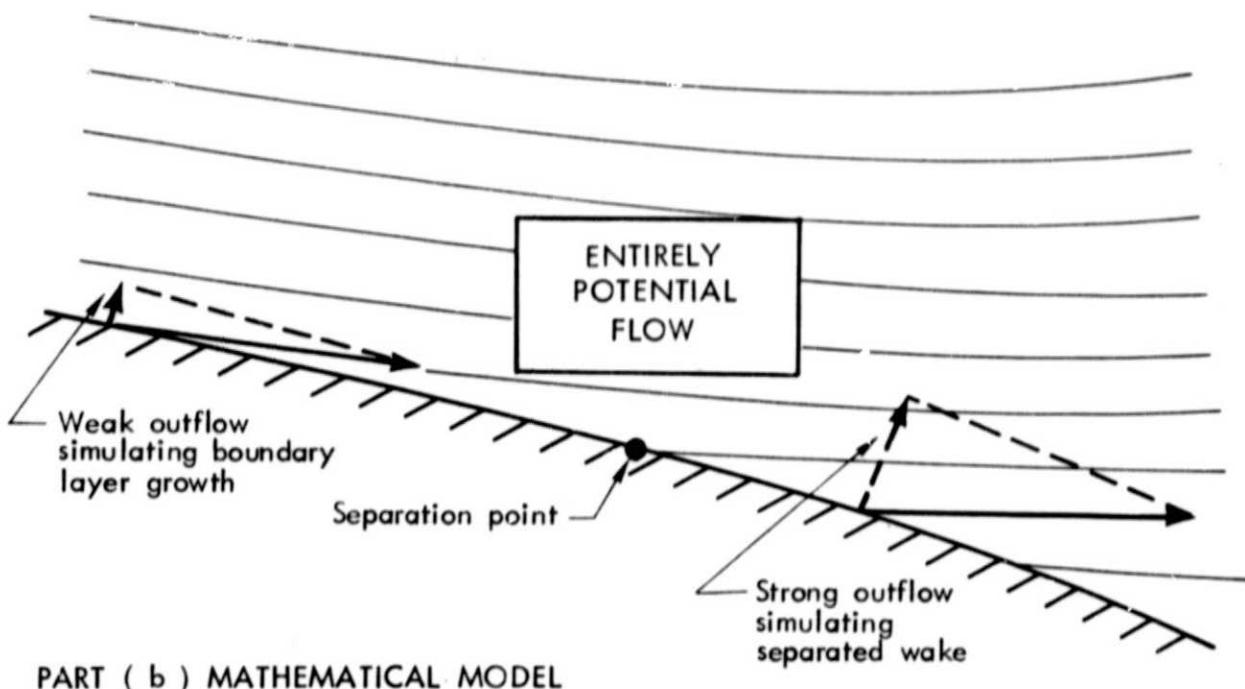
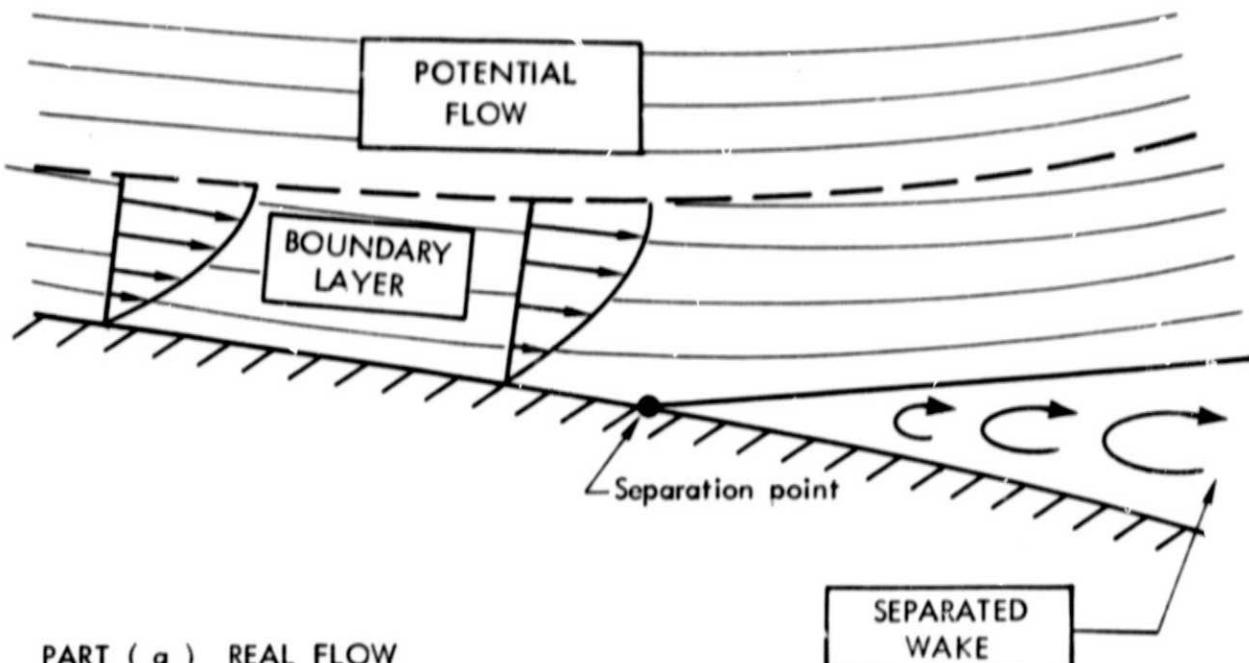


Figure 1. POTENTIAL FLOW MODEL ACCOUNTING FOR THE BOUNDARY LAYER AND SEPARATED WAKE

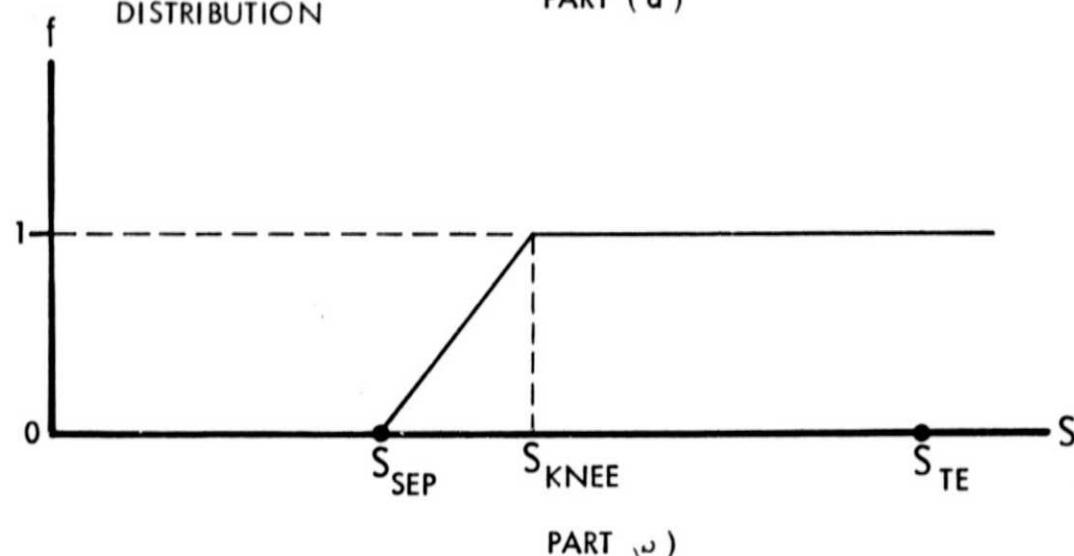
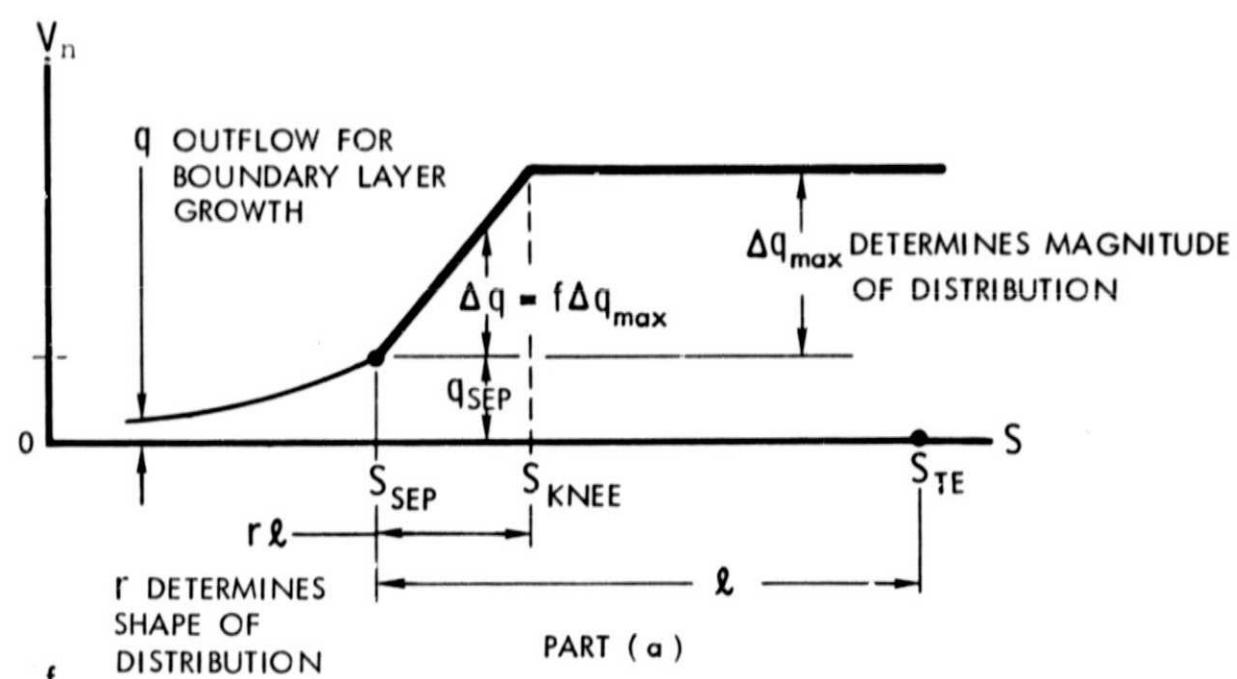
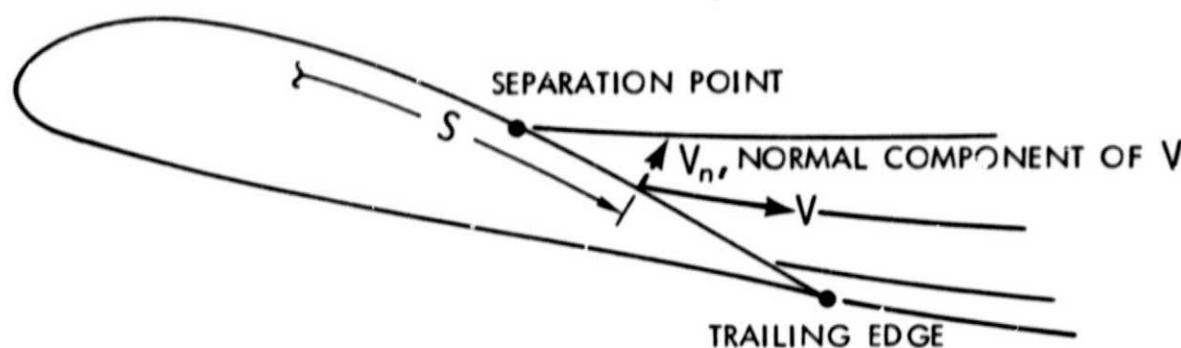


Figure 2. THE TWO-PARAMETER FAMILY OF OUTFLOW DISTRIBUTIONS FOR SEPARATION MODELING

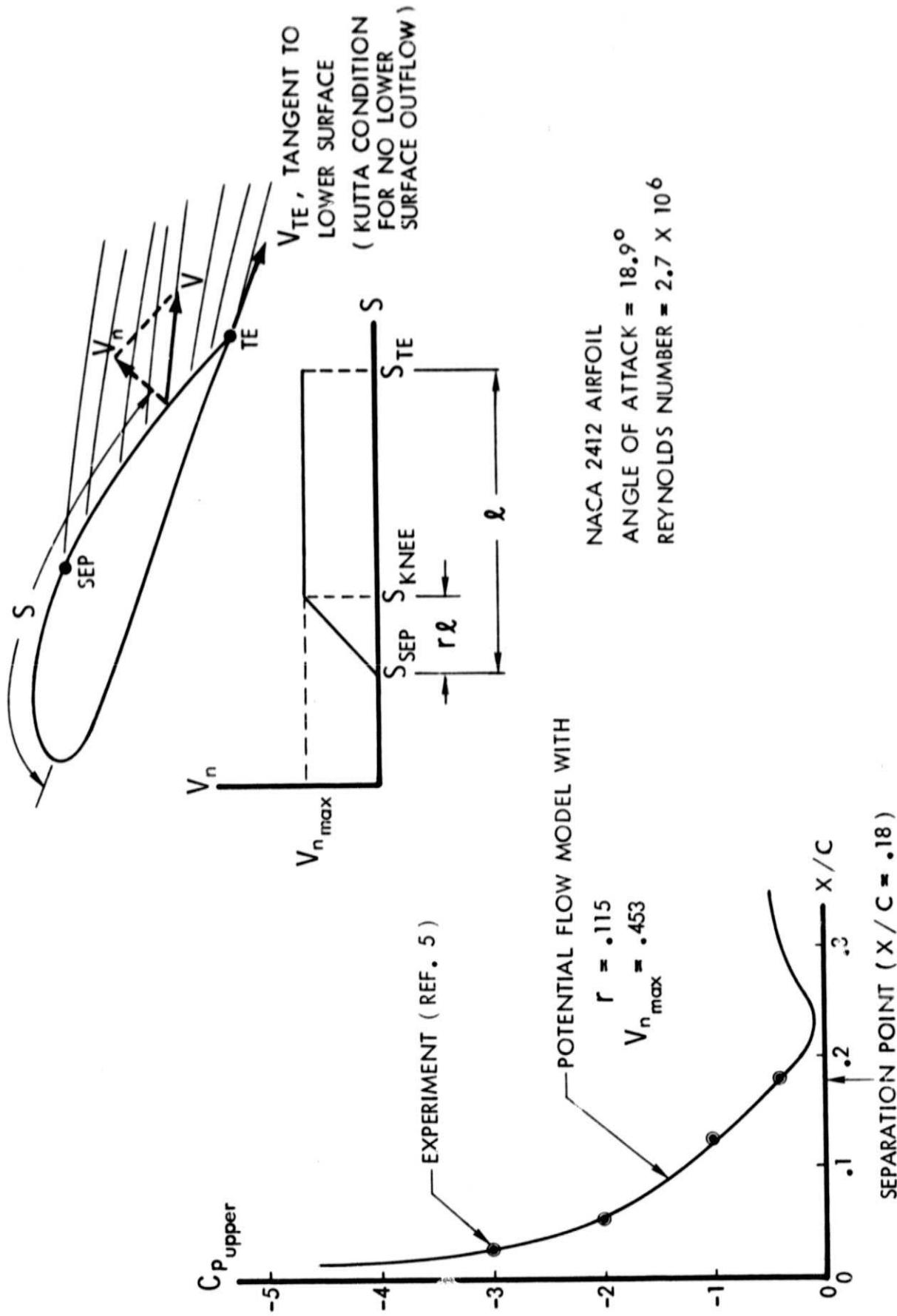


Figure 3. COMPARISON OF THE POTENTIAL FLOW SOLUTION WITH EXPERIMENT WHEN OUTFLOW IS CHOSEN TO GIVE THE BEST AGREEMENT

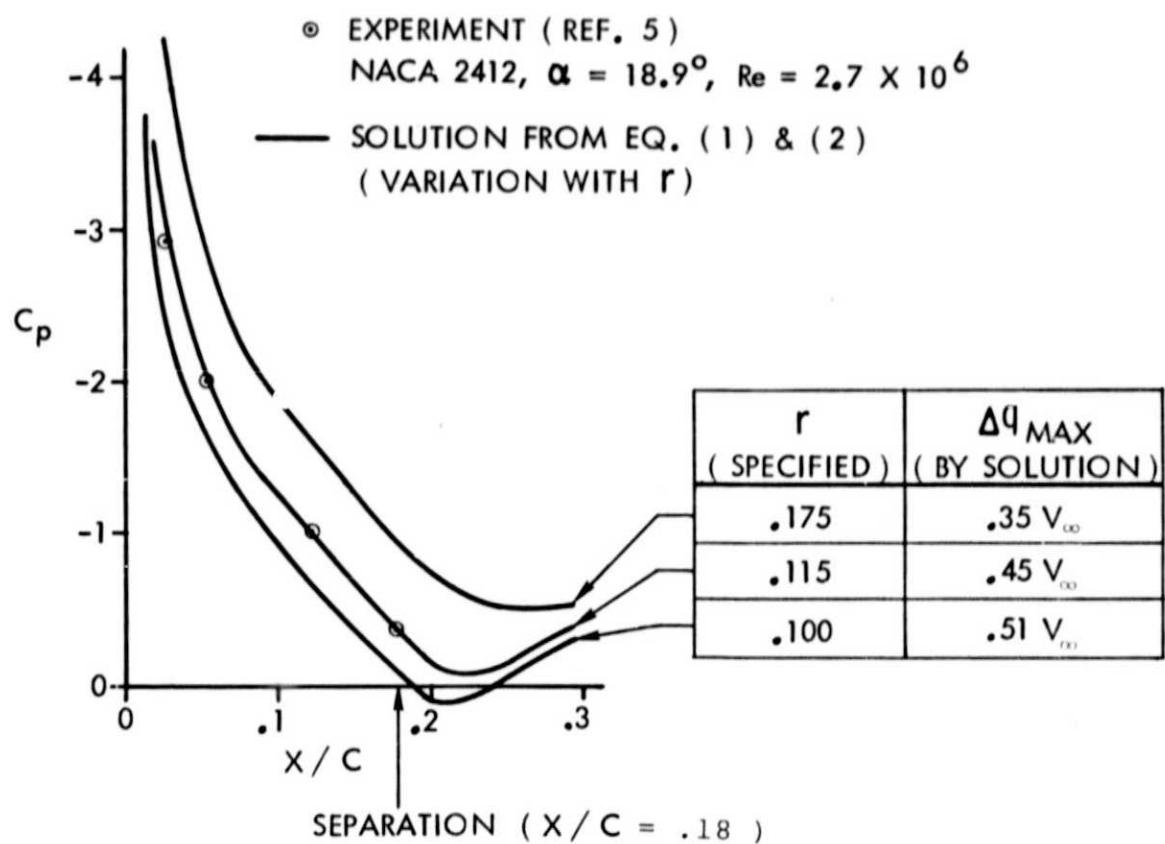


Figure 4. EFFECT OF  $r$  ON THE SOLUTION TO EQUATIONS (1) & (2)

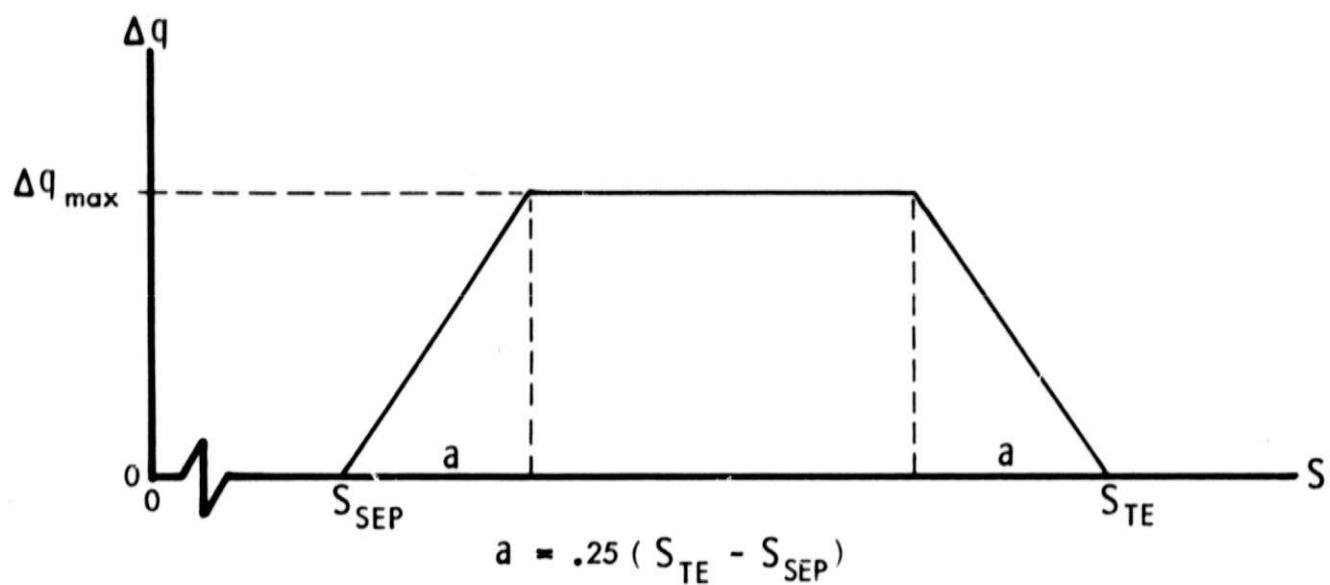


Figure 5. THE OUTFLOW DISTRIBUTION USED FOR COVE SEPARATION MODELING

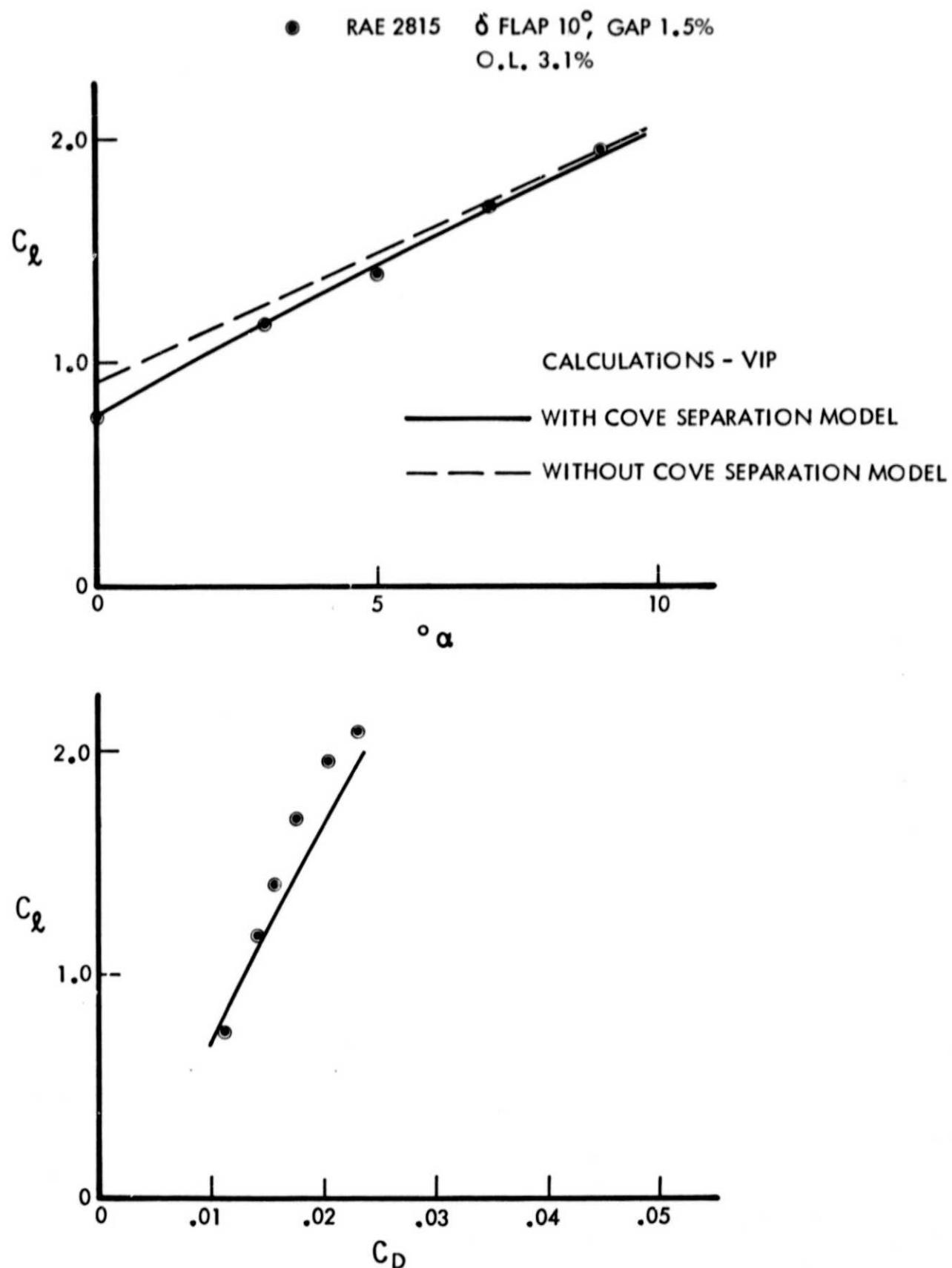


Figure 6. VIP IMPROVEMENT WITH COVE SEPARATION MODELING

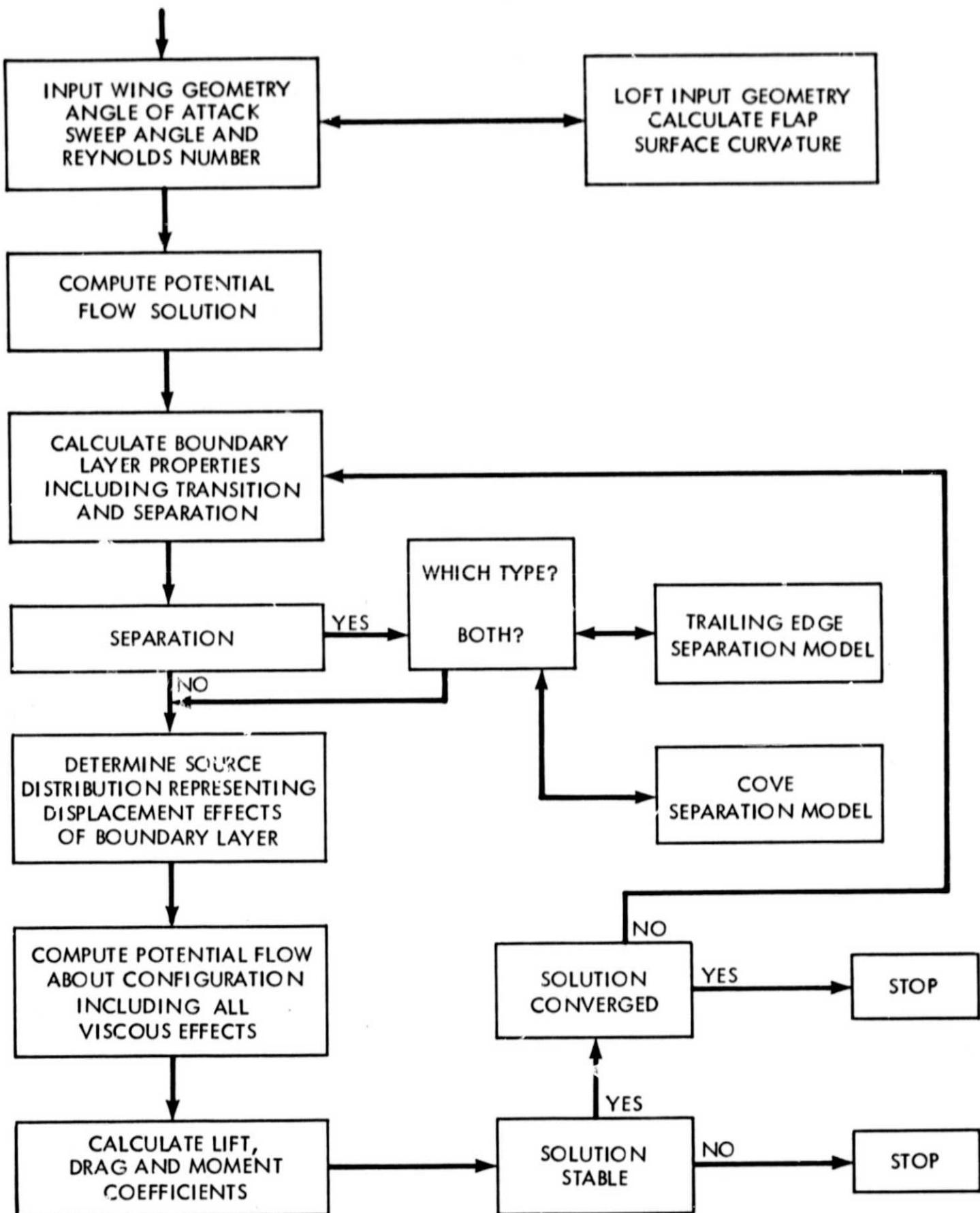


Figure 7. FLOW CHART FOR PROGRAM VIP WITH SEPARATION MODELING

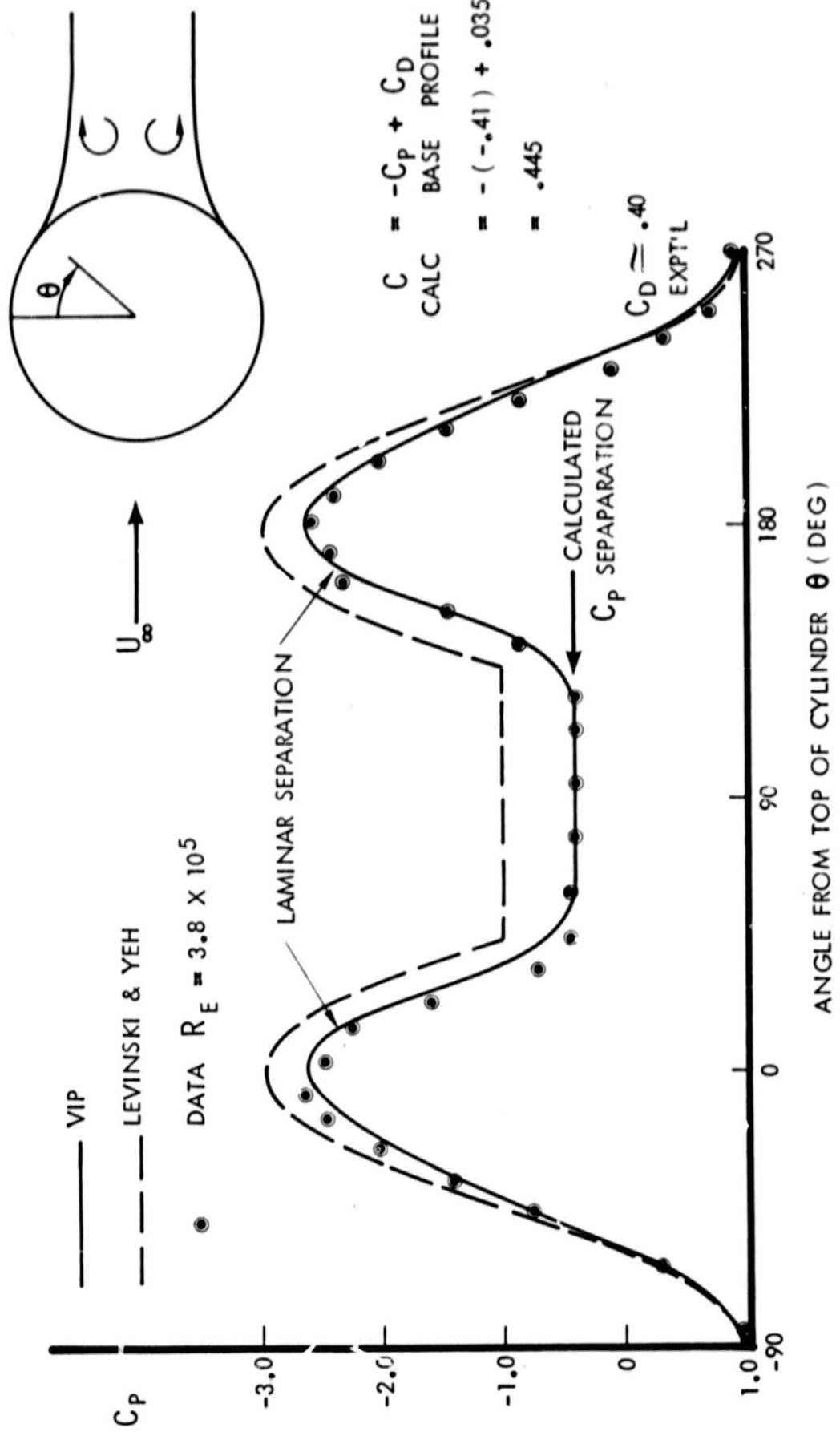


Figure 3. CIRCULAR CYLINDER WITH SEPARATION

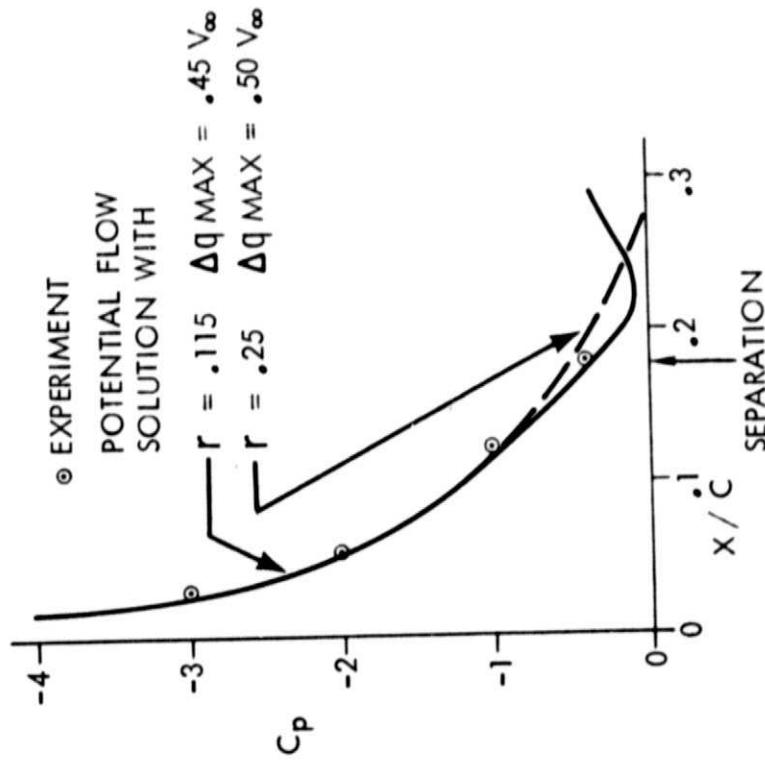


Figure 9. TWO DIFFERENT OUTFLOW DISTRIBUTIONS GIVING NEARLY THE SAME RESULTS UPSTREAM OF SEPARATION

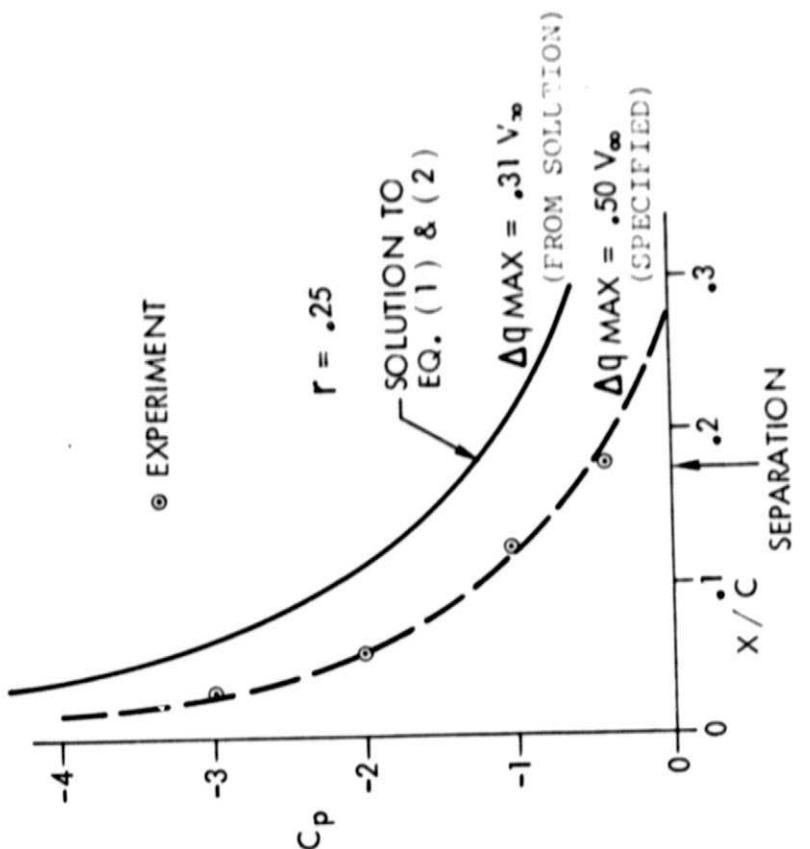


Figure 10. POTENTIAL FLOW SOLUTIONS FOR  $r = .25$ ; BEST FIT VERSUS SOLUTION TO EQUATIONS (1) & (2)